TURBINE IMPELLER AS A TANGENTIAL CYLINDRICAL JET* **

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A model is tested experimentally of a standard six-blade turbine impeller rotating in a cylindrical vessel with radial baffles under the turbulent regime of the flow. The proposed model enables the field of mean velocity and its components to be described quantitatively in the discharge stream from the rotating standard turbine impeller. The relative size of the turbine with respect to the vessel is given by $d/D = 1/4; 1/3$.

This paper is devoted to the theoretical and experimental study of the characteristics of the velocity field in the discharge flow from the blades of a rotating turbine impeller in a standard mixing vessel with radial baffles under the turbulent regime of the flow (Fig. 1). This region was selected as an object of our investigation for it differs from the rest of the batch: It is in intimate contact with the source of mechanical energy (impeller), exhibits high liquid velocities, high velocity gradients, intensive turbulence and considerable shear stresses. It is this region where the largest amount of the mechanical energy of the whole batch is being dissipated. Owing to its small volume it exhibits also maximum values of rate of energy dissipation. The hydrodynamics of this region plays a major role in affecting the process in the remaining portion of the batch as well. From the theoretical point of view the description of the hydrodynamics of the whole batch must start from the understanding of this region.

As far as the study of the velocity field in mixed systems of this type is concerned the majority of the thus far published papers are of experimental nature. Attempts for a mathematical description of the problem exist but the description of the flow is not systematic and complete as it does not reflect the relation to the geometrical arrangement of the mixed system. The investigators applied various experimental techniques for experimental research: Various modifications of the photographic method of tracers[1,2,12,18], various types of Pitot tubes[3-5,9], hot-film, or hot wire anemometers[6-10,14-15,17,32] and others. The experiments, however, were mostly carried out under hardly comparable conditions. Directly comparable experimentally determined velocity profiles are only those measured by Nielsen[1], Cutter[2], Cooper[3], Krátký[4], De Souza[5], Oldshue[6] and Günkel and Weber[32]. These authors used a similar system as the one

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used in this work to be studied also under the turbulent regime of the flow. The results indicate a bell-shaped velocity profile in the examined stream with a distinct maximum at the axis of the stream. The profile flattens with increasing radial distance from the impeller and the direction of the flow appears as radially-tangential one. However, the results of the above mentioned authors do not provide the overall picture of the flow in the studied stream and the results obtained by various experimental techniques carry an error of spatial averaging and/or do not cover the whole region of concern. Yet, they represent a fundamental information about the velocity field in the stream and have been used for comparison with the results of a systematic study presented in this work.

There exist papers in the literature in which the study of the velocity field in the stream included also the fluctuation characteristics of the velocity. The experimental techniques applied for this purpose were either a thermoanometer, hot-film or hot-wire techniques[6–10,14,15,17,32], and a photographic technique of tracers[1,2,16]. The published results on the velocity field, particularly the velocity fluctuations by the thermoanemometer technique, however, may be taken only as informative as the probe was in no case placed in the system with regard to the direction of the mean flow of liquid. The accuracy of the measurements was thus considerably restricted. Nevertheless, these results indicate that the intensity of turbulence in the examined region takes values between 20 and 80%. In more detail this problem was investigated by Nielsen[1] and Cutter[2] who used the photographic technique of tracers. In spite of its drawbacks, this technique is very illustrative and data evaluation is easy. For this reason the data from these studies were

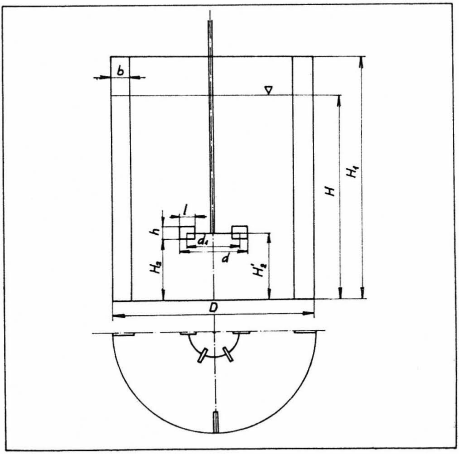

FIG. 1
Mixed System with Turbine Impeller

used to investigate the relations between the components of turbulence in the considered discharge flow.

Mathematical description of the discharge flow in the examined region of the batch was attempted by several authors[1,3–5,19]. Larsen attempted rather unsuccessfully to describe the flow in the whole batch using the concept of a two-dimensional potential flow. A first serious attempt to describe the velocity field in the stream was made by Cooper[3]. His considerations were based on mass and momentum balance of the liquid in the space delimited by the rotating impeller. The derivation of the equation for the axial profile of the radial velocity component, however, starts from incorrect assumptions and the balance of momentum does not take into consideration the principles of vector analysis. The presented profile of the radial velocity component then does not correspond to the experimentally determined profile. Krátký[4], who started from the critique of Cooper's work, derived, based on a simplification and the balance of mass, a relationship for the axial profile of the radial velocity component. Although deviating in the proximity of the outer edge of the discharge flow from the experimentally observed profile, the profile, proposed by his analysis, suits better than Cooper's model. The other cited authors used for the description of the flow in the studied region the theory of submerged jet[20–24]. Tollmien[24] proposed a model of a two-dimensional turbulent jet discharging from an infinitely long narrow slot into an infinitely large volume of liquid at rest. The solution starts from the equation of continuity and the Reynolds equation for the mean velocity of liquid under the turbulent regime of the flow. Nielsen[1] utilized the same approach. The impeller was simulated by a cylindrical tangential jet and the underlying equations were solved in cylindrical coordinates adequate to the investigated mixed system. The result of this effort was an implicit form of the axial profile of the radial velocity component and a relationship for the determination of the principal component of the turbulent shear stress. The results of Nielsen were utilized by De Souza[5]. In contrast to Nielsen, De Souza assumed free turbulence in the turbulent discharge flow and obtained an expression for the profile of the radial velocity component this time in an explicit form. This description of the flow has been successful although it still does not reflect the dependence of the four parameters of the velocity profile on the geometrical arrangement of the system.

For the experimental study of the flow in the stream we used a directional five-hole Pitot tube and adopted the earlier described[29,30] technique of measurement and pressure data processing. Because the velocity field in the turbulent flow was investigated by a anemometer with a hot-film wedge-shaped probe, the literature sources dealing with this technique had been studied. The principle of measurements and interpretation of results were described in the book by Hinze[21], Rasmussen[25,26] and Ling[27]. The interpretation of the voltage signal of the anemometer required analysis of the flow past the wire or the film of the probe. Ling applied the worked out theory of the flow past a cylinder[21] also to the flow past the current types of the wedge film probes and determined a correction factor on the unsymmetry of the flow. The true course of this unsymmetry was also investigated by Fořt and coworkers[28]. However, the available literature does not provide the interpretation of the voltage signal of the anemometer with the hot-film probe under complex hydrodynamic conditions prevailing in stream at high intensity of turbulence.

The fundamental problem was to describe the velocity field in the discharge flow using the model of De Souza[5] and set up its general form incorporating also the effect of the geometry of the system and of the frequency of revolution of the impeller. The next problem was to determine experimentally the velocity profiles, velocity fluctuations and the direction of the flow. Based on these data the above model can be quantitatively amended. In the field of experimental technique the method of detecting velocity profiles by the hot-film probe anemometer had to be modified for it accounted also for the unsymmetry of the flow past the probe in a system exhibiting high level of intensity of turbulence.

THEORETICAL

Fig. 2 depicts a part of the mixed system and the frame of reference. A point within the mixed system is set by the cylindrical coordinates z, r, α_0. The vector of mean velocity at a certain point within the batch is determined by its magnitude and the direction given by the angles α, β. The angle α is the angle between the velocity vector and its projection onto a horizontal plane passing through the measuring point. The angle β is an angle made by the above mentioned projections and the straight line passing through the measuring point in radial direction. The quantities $\bar{w}_r$, $\bar{w}_{tg}$, $\bar{w}_{ax}$ are the radial, tangential and axial components of the vector of mean velocity.

The description of the velocity field in the discharge flow from the rotating turbine impeller starts from the equation of continuity and the Reynolds equation[21,31] for the mean velocity under the turbulent regime. It is assumed that the turbine impeller as a source of motion may be simulated by an axially symmetric cylindrical slot – termed the tangential cylindrical jet. The slot is shown in Fig. 3 with the radius designated by a. The liquid discharging from the tangential cylindrical jet has the only non-zero velocity component in the tangential direction and discharges into the liquid of infinite extent and at rest. Further we introduce additional simplifying assumptions:

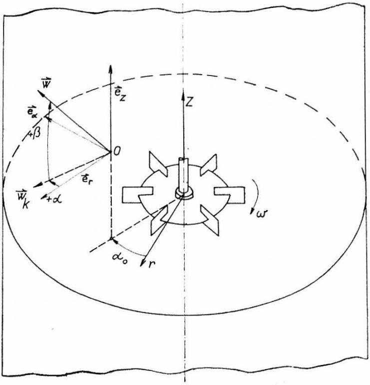

FIG. 2

Part of Mixed System and Frame of Reference

1) The process in question is quasi-stationary. *2*) The liquid is incompressible. *3*) The stream is a turbulent three-dimensional jet symmetrical about the axis of revolution of the impeller. Thus no variation of the examined quantities takes place along the coordinate α_0. *4*) The liquid is a Newtonian one with constant viscosity, while the viscosity forces are negligible compared to the outher forces acting in the jet. *5*) Pressure changes are due to the hydrostatic pressure. They exist only along the z axis and do not affect the regime of the flow. *6*) The components of the tensor of turbulent stresses and their derivatives with respect to the coordinates r, α_0, z are negligible compared to the components $\bar{\tau}_{r,ax}$, $\bar{\tau}_{tg,ax}$ and their derivatives with respect to the axial coordinate z. *7*) The liquid discharges from a cylindrical tangential jet as a stream (Fig. 3) with only the free turbulence existing within the stream[20]. The turbulent stress is then expressed according to the second Prandtl's hypothesis by

$$\bar{\tau} = \varrho k s |\bar{w}_{max} - \bar{w}_{min}| \, \partial \bar{w} / \partial z \,, \qquad (1)$$

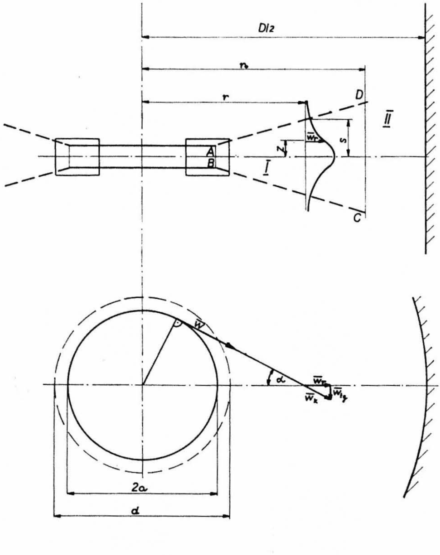

FIG. 3
Cylindrical Tangential Jet

where $\bar{\tau}$ is the resulting shear stress, ϱ is the density of the charge, s is the half-width of the stream and k is a coefficient. *8)* The velocity profiles along various radii are similar and may be expressed by[21-23]

$$\bar{w}/\bar{w}_{max} \quad \text{resp.} \quad \bar{w}_r/\bar{w}_{rmax} = f(\eta)\,, \tag{2}$$

where

$$\eta = z/s\,, \quad s = r/\sigma\,.$$

The symbols in these equations have the following meanings:

z the axial coordinate, r the radial coordinate, s the half-width of the stream and σ is a coefficient. *9)* The velocity component $\bar{w}_{ax}$ is by an order of magnitude smaller than the components $\bar{w}_r$, $\bar{w}_{tg}$. *10)* The following boundary conditions apply:

$$\text{for} \quad z = 0: \qquad \bar{w}_{ax} = 0\,, \quad \bar{w}_r = \bar{w}_{rmax}\,, \quad \partial\bar{w}_r/\partial z = 0\,, \tag{3}$$
$$\text{for} \quad z \to \pm\infty: \quad \bar{w}_r = 0\,, \quad \partial\bar{w}_r/\partial z = 0\,.$$

The equations of motion and continuity transform under the above listed conditions from the general form in the cylindrical coordinates[21,5] into the form[32]

$$\bar{w}_r \frac{\partial \bar{w}_r}{\partial r} + \bar{w}_{ax} \frac{\partial \bar{w}_r}{\partial z} - \frac{a^2}{r^2(r^2 - a^2)}\, \bar{w}_r^2 = \frac{\sqrt{(r^2 - a^2)}}{\varrho r} \frac{\partial \bar{\tau}}{\partial z} \tag{4a}$$

$$\bar{w}_r \frac{\partial \bar{w}_r}{\partial r} + \bar{w}_{ax} \frac{\partial \bar{w}_{ax}}{\partial z} + \bar{w}_r^2 \left(\frac{1}{r} - \frac{r}{r^2 - a^2}\right) = \frac{\sqrt{(r^2 - a^2)}}{\varrho r} \frac{\partial \bar{\tau}}{\partial z} \tag{4b}$$

$$-\frac{\partial p}{\partial z} = \varrho \boldsymbol{g}\,, \tag{4c}$$

$$\frac{1}{r} \frac{\partial (r\bar{w}_r)}{\partial r} + \frac{\partial}{\partial z}(r\bar{w}_{ax}) = 0\,. \tag{5}$$

In the search for an expression for the profile of the radial velocity component it suffices to solve a single equation of motion, either (*4a*) or (*4b*), using Eq. (*1*) and (*5*). In the solution $\bar{w}_{min}$ is neglected compared to $\bar{w}_{max}$ and the kinematic turbulent viscosity, ε, is defined as

$$\varepsilon = ks\bar{w}_{max}\,. \tag{6}$$

After some manipulation[5,22,23,32] and introduction of the stream function the partial diffeerential equation (*4a*) or Eq. (*4b*) can be put, with the aid of the above assumptions, into the form of an ordinary differential equation. Its solution yields a relation for the radial velocity component in dependence on the axial coordinate in the form

$$\bar{w}_r = \frac{A}{2}\left(\frac{\sigma}{r^3}\right)^{1/2} (r^2 - a^2)^{1/4} \left[1\text{-tg}\, h^2(\eta/_2)\right], \tag{7}$$

where we can define

$$\bar{w}_{r\max} = \frac{A}{2}\left(\frac{\sigma}{r^3}\right)^{1/2} (r^2 - a^2)^{1/4}. \tag{7a}$$

Introducing dimensionless velocity and dimensionless axial coordinate by[4,32]

$$\bar{W}_r = \bar{w}_r/\pi dn\,, \quad Z = 2z/h \tag{8a,b}$$

and expressing the shift of the velocity profile off the horizontal plane of symmetry of the impeller, an equation for the radial velocity component is obtained in the form:

$$\bar{W}_r = A_1\{1 - \text{tg}\, h^2[A_2(Z - A_3])\}. \tag{9}$$

The parameter A_1 represents maximum radial velocity component of liquid on a given profile normalized by the peripheral velocity of the outer edge of the blades of the impeller. A_1 can be also expressed as

$$A_1 = \frac{A}{2\pi dn}\left(\frac{\sigma}{r^3}\right)^{1/2} (r^2 - a^2)^{1/4}. \tag{9a}$$

The parameter A_2 is associated with the width of the stream in a given position normalized by the width of impeller's blade h, *i.e.*

$$A_2 = \sigma h/4r\,. \tag{9b}$$

The parameter A_3 in Eq. (*9*) expresses the dimensionless axial coordinate of the velocity maximum, *i.e.* its shift off the horizontal plane of symmetry of the impeller. It is obvious that the parameters A_1, A_2, A_3 are functions of both the geometrical arrangement of the mixed system and the radial distance of the examined point from the tangential cylindrical jet. These parameters and their dependences may be evaluated from experimental data by a suitable computational method. For this purpose a dimensionless radial distance, R, has been introduced, based on the following reasoning:

Let us start from the concept of the cylindrical jet shown in Fig. 3. Fig. 4 shows schematically the case when two cylindrical tangential jets of radii a_1, a_2, discharge liquid that moves at two points on the radii r_1 and r_2 in the same direction. For the angles α_1 and α_2 at r_1 and r_2 we then may write

$$\alpha_1 = \alpha_2 \,. \tag{10}$$

The peripheral velocity of the rotating tangential jet (the outer edge of the blades of the impeller) is proportional to the diameter of the jet at the same frequency of revolution. The paths of liquid particles from the source toward the measuring point are then also in the ratio of the radii of the jets. It may be thus assumed that under such conditions the velocity at the two measuring points, given by the coordinates r_1, r_2, will be the same just like its radial and tangential components. Then we may write

$$\sin \alpha_1 = \sin \alpha_2 \,, \tag{11}$$

when

$$\sin \alpha_1 = a_1/r_1 = c/(r_1 - a_1) \,, \tag{12}$$

$$\sin \alpha_2 = a_2/r_2 = b/(r_2 - a_2) \,. \tag{13}$$

From the foregoing relations and by inspection of Fig. 4 it follows

$$(r_1 - a_1)/r_1 = (r_2 - a_2)/r_2 \,. \tag{14}$$

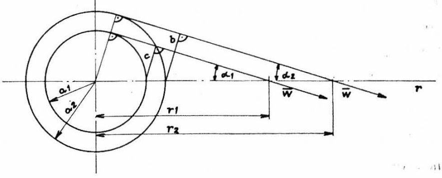

FIG. 4
Two Tangential Jets—Dimensionless Radial Distance

Based on this finding the dimensionless radial distance, R, was defined by

$$R = (r - a)/r. \tag{15}$$

The parameters of the profile of the radial velocity component, A_1, A_2, are functions of the radial coordinate. These functions can be taken in the linear form

$$A_1 = a_1 + b_1 R \tag{16}$$

and

$$A_2 = a_2 + b_2 R . \tag{17}$$

The parameter A_3 also depends on the radial coordinate, but, in addition, also on the height of the impeller over the bottom of the mixing vessel, H'_2. Let us assume that this relation can be also fitted by the following proportionalities

$$A_3 = b_3 R \tag{18}$$

and

$$b_3 = b' H'_2/H , \tag{19}$$

where for

$$H'_2/H = 1/2 : \quad b' \neq 0 ,$$

and for

$$H'_2/H < 1/2 : \quad b' = 0 .$$

The proposed model should enable an estimate of the velocity in an arbitrary point of the stream from the knowledge of the radius of the tangential cylindrical jet and the parameters a_1, b_1, a_2, b_2 and b provided the height of the impeller over the bottom satisfies $H'_2/H > 1/2$. Using this model one can easily derive[32] expressions for the total volumetric flow rate $\dot{V}$ through a selected surface on an arbitrary radial coordinate, the pumping capacity of the turbine impeller, $\dot{V}_t$, appropriate flow rate criteria K_T and K_t and finally for the dimensionless stream function ψ (Table I).

EXPERIMENTAL

In order to test the proposed theoretical model and evaluate its parameters in dependence on position in the mixed system, the geometrical arrangement and the frequency of revolution of the impeller, the mean velocity and the direction of the flow were measured. These data were supplemented also by measurements of the velocity fluctuations[32]. The direction of the mean velocity of the flow was assessed from the in-advance calibrated directional five-hole Pitot tube.

The velocity and its fluctuations were evaluated from measurements by a calibrated wedge-shaped hot-film probe of the anemometer using a DISA instrumentation[32]. An analog MEDA computer was used to process the signal of the anemometer.

The experiments were carried out in a vertical cylindrical vessel with baffles. The rotating impeller was located axially with the ratio of the height of the liquid level and the vessel diameter being $H/D = 1$. The number of baffles was 4, their width, b, equalling 0·1 D. The diameter of the vessel was $D = 1$ m. The impeller was a disc turbine and six perpendicular blades in the arrangement obeying Czechoslovak standard ČSN ON 691021[18]. The characteristics of the velocity field were measured in the following radial distances R: 0·170; 0·208; 0·250; 0·290; 0·333 m for $d =$

TABLE I

Characteristics of the Velocity Profile in the Discharge Flow from the Rotating Turbine Impeller
Mean direction of flow: $\beta = 0$, $\alpha = \arcsin a/r$.

Interpretation of axial profile I	Interpretation II
Mean velocity	
$\overline{W}_r = A_1\{1 - \mathrm{tgh}^2\,[A_2(Z - A_3)]\}$	$\overline{W}_r = \dfrac{A}{2\pi dn}\left(\dfrac{\sigma}{r^3}\right)^{1/2}(r^2 - a^2)^{1/4}\,.\,\left\{1 - \mathrm{tgh}^2\left[\dfrac{\sigma}{4r}(Z - A_3)\right]\right\}$
$\overline{W}_{tg} = \overline{W}_r\,\mathrm{tg}\,\alpha$	
$\overline{W}_{ax} = 0$	
$\overline{W} = \sqrt{(\overline{W}_r^2 + \overline{W}_{tg}^2)}$	
Volumetric flow rate, criterion of total flow rate	
$\dot{V}_T = \pi^2\,ndhr\,(A_1/A_2)$	$\dot{V}_T = 2\pi A\left(\dfrac{r}{\sigma}\right)^{1/2}(r^2 - a^2)^{1/4}$
$K_T = \dfrac{\pi^2 hr}{d^2}(A_1/A_2)$	$K_T = \dfrac{2\pi A}{nd^3}\left(\dfrac{r}{\sigma}\right)^{1/2}(r^2 - a^2)^{1/4}$
Dimensionless stream function	
$\Psi = \dfrac{\pi hr}{2d^2}\dfrac{A_1}{A_2}\,\mathrm{tgh}\,[A_2(Z - A_3)]$	$\Psi = \dfrac{A}{nd^3}\left(\dfrac{r}{\sigma}\right)^{1/2}(r^2 - a^2)^{1/4}\,.\,\mathrm{tgh}\left[\dfrac{\sigma h}{4r}(Z - A_3)\right]$

$= 0{\cdot}33$ m *i.e.* $d/D = 1/3$ and R: 0·128; 0·170; 0·208; 0·250; 0·290; 0·333 m for $d = 0{\cdot}25$, *i.e.* $d/D = 1/4$.

The distances betweem individual measuring points were put equal 4 to 9 mm in order that each axial profile contain on average 15 measured data.

RESULTS AND DISCUSSION

The direction of the flow of liquid in individual points of the discharge flow is set by the angles α and β. The fundamental finding following from the measurements of the direction of the mean flow is that the direction is predominantly radially-tangential while with increasing distance from the source of motion the stream turns more into radial direction. The angle β thus reaches relatively low values and only rarely exceeds ± 15 degrees. The situation is illustrated in Fig. 5. In contrast, the angle α as a consequence of the just mentioned tendency is large in the proximity of the source of motion falling into the interval $\alpha \in \langle 40; 60 \text{ deg.}\rangle$. Its value again decreases in the direction away from the axis of the stream due to the contact with the ambient liquid. With increasing radial coordinate r, *i.e.* with increasing distance from the source of motion, α steadily decreases and the profiles flattens to level off to almost a constant value. The average values of the angle, $\bar{\alpha}$, over the profile with increasing radial distance decrease from about 53 degrees to about 17 degrees. This situation is well illustrated in Fig. 6. The experimentally found directions of the

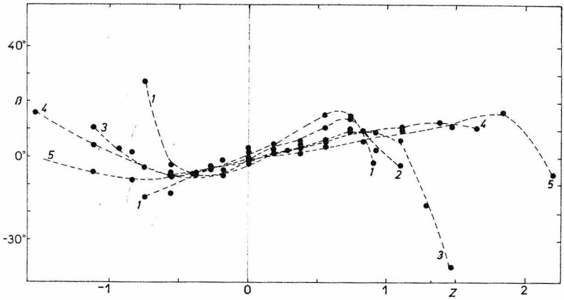

FIG. 5

Typical Direction of Flow Profiles Characterized by Angle β ($d/D = 1/3$; $n = 90\ \text{min}^{-1}$)

R	0·188	0·337	0·448	0·524	0·586
Curve	*1*	*2*	*3*	*4*	*5*

flow thus confirm the concept simulating the turbine impeller by a cylindrical tangential jet (see also the text refering to Fig. 4). This also confirms assumption *8* neglecting the axial velocity component with respect to the remaining components.

The results of the performed experiments are profiles of mean velocity, its components – the radial, tangential or axial – in the discharge flow of liquid. The results reveal the character of these profiles exhibiting a distinct maximum at the axis of the stream (this situation is illustrated by the profiles of the dimensionless radial velocity component in Fig. 7). Further it can be observed gradual flattening of the profiles with increasing radial coordinate. A marked shift of the mentioned maximum from the horizontal plane of symmetry of the impeller upwards is a consequence of the off-symmetry position of the impeller within the system, characterized by the magnitude of the simplex $H'_2/H = 1/3$. It cannot be left unnoticed that the discharge flow widens with increasing distance from the impeller. This is induced by the turbulent transfer of momentum in the vertical direction. The profiles of the tangential velocity component markedly flatten with increasing radial coordinate with simultaneous decrease of the magnitude of this component. This is fully compatible with the concept of cylindrical tangential jet. The axial velocity component remains very small, confirming assumption *8*. The profiles of the angle α, indicating the direction of the mean flow, were processed to give a versatile value for the parameter of the velocity profile, a, depending on the size of the impeller and the direction of the

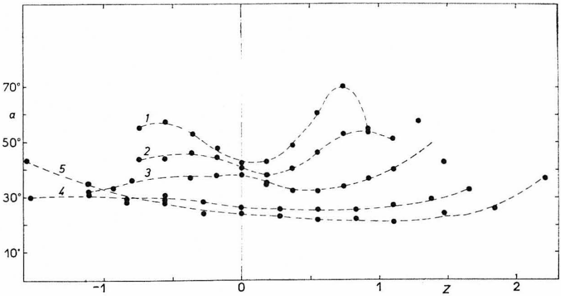

FIG. 6

Typical Experimental Direction of Flow Profiles Characterized by Angle α ($d/D = 1/3$; $n = 90 \text{ min}^{-1}$)

R	0·188	0·337	0·448	0·524	0·586
Curve	*1*	*2*	*3*	*4*	*5*

mean flow (see Eq. (*12*) and Fig. 3 and 4). The versatile value of a as well as those of the parameters of other models are: $a = 0{\cdot}34$ d; $A_1 = 0{\cdot}81(1 - R)$; $A_2 = 2{\cdot}63$ to $3{\cdot}26R$; $A_3 = b_3R$; $b_3 = 0$ for $H'_2/H = 1/2$; $b_3 = 0{\cdot}74$ for $H'_2/H < 1/2$; $\sigma = 11{\cdot}2$; $A/(2\pi\, dn) = 0{\cdot}35$ m). The parameters of the equation (*9*) for the axial profiles of the radial velocity component were evaluated from the experimental profiles by nonlinear

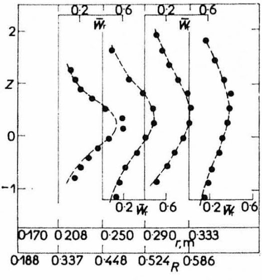

FIG. 7

Typical Experimental Profiles of Dimensionless Radial Mean Velocity Component $\overline{W}_r$ in the Discharge Flow from Turbine Impeller ($d/D = 1/3$; $n = 90\ \text{min}^{-1}$)

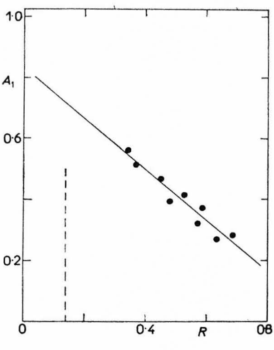

FIG. 8

Parameter A_1 as a Function of Dimensionless Radial Coordinate R ($d/D \in \langle 1/4;\ 1/3 \rangle$)

Broken line shows radial coordinate of outer edges of the blades of the turbine impeller.

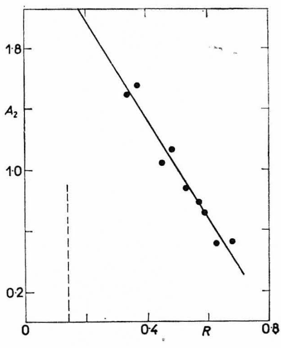

FIG. 9

Parameter A_2 as a Function of Dimensionless Radial Coordinate R ($d/D \in \langle 1/4;\ 1/3 \rangle$)

Broken line shows coordinate of outer edges of the blades of the turbine impeller.

regression analysis. For these parameters we then evaluated their dependence on position (radial coordinate $\boldsymbol{R}$), in the mixed system, geometrical arrangement (distance of the impeller over the bottom, or the simplex H'_2/H) and on the frequency of revolution of the impeller. The dependence of the parameters of the dimensionless radial velocity component profiles, A_1, A_2, A_3, on the dimensionless radial coordinate R is shown in Figs 8–10. Solid lines indicate regressed relations. The straight lines in Fig. 10 represent regressed linear dependences *a*) in the general form and *b*) without the absolute term. Broken lines indicate radial distance corresponding to the radius of the impeller. The same analysis[32,33] was applied to the data published by Cooper[3], Nielsen[1], De Souza[5], Krátký and Cutter[2]. The authors carried out their measurements in vessels of various diameters $D \in \langle 0{\cdot}28\ \text{m};\ 1{\cdot}0\ \text{m}\rangle$ and various relative diameters of the impellers $d/D \in \langle 0{\cdot}2;\ 0{\cdot}4\rangle$. In their experiments the frequency of revolution of the impellers was varied between $n \in \langle 50\ \text{min}^{-1};\ 600\ \text{min}^{-1}\rangle$ and the velocity field was studied at various radial positions. The impellers were located at heights over the bottom characterized by the values of the simplex $H'_2/H = 1/2$, $1/3$ and $1/4$. Table I presents the model of the flow in the investigated region offering two alternatives for interpretation of the axial profile of the radial

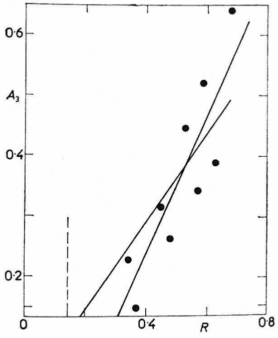

FIG. 10

Parameter A_3 as a Function of the Dimensionless Radial Coordinate R ($d/D \in \langle 1/4;\ 1/3\rangle$)

Broken line shows radial coordinate of outer edges of the blades of the turbine impeller.

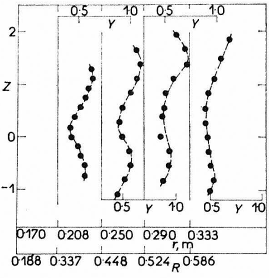

FIG. 11

Typical Experimental Intensity of Turbulence Profiles $Y = (\overline{w'^2} + \overline{u'^2})^{1/2}/\overline{w}$ in the Discharge Flow from the Rotating Blades of the Turbine Impeller ($d/D = 1/3$; $n = 90\ \text{min}^{-1}$)

velocity component. In part I the shown parameters take the form of a dependence on position within the system and height of the impeller over the bottom of the vessel. In part II the parameters possess mostly a universal character. From the theoretical derivation there follow also additional quantities characterizing the flow of liquid in the discharge flow. The results of this work also incorporate determination of the corresponding profiles of intensity of turbulence. The values of this quantity fall into the following intervals: $(\overline{w'^2} + \overline{w'^2})^{1/2}/\overline{w} \in \langle 30\%; 100\% \rangle$. Typical example is shown in Fig. 11. A part of the processing procedure was in all cases also the estimation of error of the evaluated quantities. Based on these data it can be concluded that the estimate of the parameters of the proposed model suffices for technical purposes.

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LIST OF SYMBOLS

A	universal parameter of radial velocity component profile, $m^2 s^{-1}$
A_1	universal parameter of radial velocity component profile
A_2	universal parameter of radial velocity component profile
A_3	universal parameter of radial velocity component profile
a	radius of cylindrical tangential jet, m
a_1	constant
a_2	constant
b	width of baffle, m
b_2	constant
b_3	constant
b'	constant
D	diameter of vessel, m
d	diameter of impeller, m
d_1	diameter of turbine disc, m
f	similarity function
H	height of liquid over bottom, m
H_2	height of lower edge of blades over bottom, m
H_2'	height of turbine disc over bottom, m
h	width of impeller's blade, m
K_T	criterion of total flow rate
K_t	criterion of impeller pumping flow
k	coefficient, m
l	length of blade of impeller, m
n	frequency of revolution of impeller, s^{-1}
R	dimensionless radial distance
r	radial coordinate, m
s	half-width of liquid stream, m
$\dot{V}_T$	total volume tric flow rate, $m^3 s^{-1}$
$\dot{V}_t$	pumping capacity of impeller, $m^3 s^{-1}$
u'	fluctuation velocity in direction perpendicular to mean velocity, $m s^{-1}$

$\overline{w}$ mean velocity, $m\ s^{-1}$
w' fluctuation velocity in direction of mean velocity, $m\ s^{-1}$
$\overline{W} = \overline{w}/\pi\ dn$ dimensionless mean velocity
z axial coordinate, m
Z dimensionless axial coordinate
α angle, deg
β angle, deg
Ψ dimensionless stream function
σ universal parameter of radial velocity component profile
ε eddy kinematic viscosity, $m^2\ s^{-1}$
$\bar{\tau}$ turbulent shear stress, $N\ m^{-2}$
α_0 tangential (angular) coordinate of mixed system
ϱ density, $kg\ m^{-3}$
η dimensionless axial coordinate

Subscript

ax axial
r radial
tg tangential

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